AN

ESSAY

ON THE

USEFULNESS

OF

Mathematical Learning.

IN A

LETTER

FROM

A GENTLEMAN in the *CITY*, to his
FRIEND at *OXFORD*.

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RO. MANDER,

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E S S A Y

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USEFULNESS

O F

MATHEMATICAL LEARNING, &c.

S I R,

I A M glad to hear from you, that the Study of the *Mathematics* is Promoted and Encouraged among the *Youth* of your *Univerſity*. The great Influence, which theſe *Sciences* have on Philoſophy, and all uſeful Learning, as well as the Concerns of the Public, may ſufficiently recommend them to your Choice and Conſideration: And the particular Advantages, which You of that Place enjoy, give Us juſt Reaſon to expe� from You a ſuitable Improvement in them. I have here ſent you ſome ſhort Refleɡions upon the *Uſefulneſs* of *Mathematical Learning*, which may ſerve as an Argument to incite you to a cloſer and more vigorous Purſuit of it.

In all Ages and Countries, where Learning hath prevailed, the *Mathematical Sciences* have been looked upon as the moſt conſiderable Branch of it. The very Name Μάθησις implies no leſs ; by which they were called, either for their Excellency ; or becauſe, of all the *Sciences*, they were firſt taught ; or becauſe they were judged to comprehend πάντα τὰ

τὰ Μαθήματε. And, amongſt thoſe, that are commonly reckoned to be the Seven Liberal Arts, Four are Mathematical, to wit, *Arithmetic, Muſic, Geometry,* and *Aſtronomy.*

But, notwithſtanding their Excellency and Reputation, they have not been taught nor ſtudy'd ſo univerſally, as ſome of the reſt; which I take to have proceeded from the following Cauſes: The *Averſion of the greateſt Part of Mankind to ſerious Attention, and cloſe Arguing*; Their *not comprehending ſufficiently the Neceſſity, or great Uſefulneſs, of theſe in other Parts of Learning*; An *Opinion that this Study requires a particular Genius and Turn of Head, which few are ſo happy as to be born with*; And *the Want of public Encouragement, and able Maſters.* For theſe, and perhaps ſome other Reaſons, this Study hath been generally neglected, and regarded only by ſome few Perſons, whoſe happy Genius and Curioſity have prompted them to it, or who have been forced upon it by its immediate Subſerviency to ſome particular Art or Office.

Therefore I think I cannot do better Service to Learning, Youth, and the Nation in general, than by ſhewing, *That the Mathematics, of all Parts of human Knowlege, for the Improvement of the Mind, for their Subſerviency to other Arts, and their Uſefulneſs to the Commonwealth, deſerve moſt to be encouraged.* I know a Diſcourſe of this Nature will be offenſive to ſome, who, while they are ignorant of Mathematics, yet think themſelves Maſters of all valuable Learning: But their Diſpleaſure muſt not deter me from delivering an uſeful Truth.

The Advantages which accrue to the Mind by Mathematical Studies, conſiſt chiefly in theſe Things: 1ſt, In accuſtoming it to *Attention.* 2dly, In giving it a Habit of *cloſe* and *demonſtrative Reaſoning.* 3dly, In freeing it from *Prejudice, Credulity,* and *Superſtition.*

Firſt,

Firſt, the Mathematics make the Mind ʼatten-
tive to the Objeƈts, which it conſiders. This they
do by entertaining it with a great Variety of Truths,
which are delightful and evident, but not obvious.
Truth is the ſame thing to the Underſtanding, as
Muſic to the Ear, and Beauty to the Eye. The Pur-
ſuit of it does really as much gratify a natural Faculty
implanted in us by our wiſe Creator, as the pleaſing
of our Senſes : Only in the former Caſe, as the Ob-
jeƈt and Faculty are more ſpiritual, the Delight is
the more pure, free from the Regret, Turpitude,
Laſſitude, and Intemperance, that commonly attend
ſenſual Pleaſures. The moſt Part of other Sciences
conſiſting only of probable Reaſonings, the Mind
has not where to fix ; and, wanting ſufficient Prin-
ciples to purſue its Searches upon, gives them over
as impoſſible. Again, as in *Mathematical Inveſtiga-
tions* Truth may be found, ſo it is not always *obvious :*
This ſpurs the Mind, and makes it diligent and at-
tentive. *In Geometria* (ſays *Quinƈtilian, lib.* I. *cap.*
10.) *partem fatentur eſſe utilem teneris ætatibus : agitari
namque animos, atque acui ingenia, & celeritatem per-
cipiendi venire inde concedunt.* And *Plato* (in *Repub.
lib.* VII.) obſerves, that the Youth, who are furniſhed
with *Mathematical* Knowlege, are prompt and quick
at all other *Sciences*, εἰς πάντα τὰ Μαθήμαϊα ὀξεῖς φαίνονϊαι.
Therefore he calls it κᾳϊὰ παιδείαν ὁδόν. And, indeed,
Youth is generally ſo much more delighted with
Mathematical Studies, than with the unpleaſant
Taſks, that are ſometimes impoſed upon them, that
I have known ſome reclaimed by them from Idle-
neſs, and Negleƈt of Learning ; and acquire in time
a Habit of Thinking, Diligence, and Attention ;
Qualities, which we ought to ſtudy by all means to
beget in their deſultory and roving Minds.

The Second Advantage, which the Mind reaps
from *Mathematical* Knowlege, is a Habit of *clear,
demonſtrative,* and *methodical* Reaſoning. We are

contrived

contrived by Nature to learn by Imitation more than by Precept : And, I believe, in that refpect, Reafoning is much like other inferior Arts (as Dancing, Singing, &c.) acquired by Practice. By accuftoming ourfelves to reafon clofely about Quantity, we acquire a Habit of doing fo in other things. It is furprifing to fee, what fuperficial inconfequential Reafonings fatisfy the moft Part of Mankind. A Piece of Wit, a Jeft, a Simile, or a Quotation of an Author, paffes for a mighty Argument: With fuch things as thefe are the moft Part of Authors ftuffed ; and from thefe weighty Premifes they infer their Conclufions. This Weaknefs and Effeminacy of Mankind in being perfuaded where they are delighted, have made them the Sport of Orators, Poets, and Men of Wit. Thofe *lumina orationis* are indeed very good Diverfion for the Fancy, but are not the proper Bufinefs of the Underftanding ; and where a Man pretends to write on abftract Subjects in a fcientifical Method, he ought not to debauch in them. Logical Precepts are more ufeful, nay, they are abfolutely neceffary, for a Rule of formal Arguing in public Difputations, and confounding an obftinate and perverfe Adverfary, and expofing him to the Audience or Readers. But, in the Search of Truth, an Imitation of the Method of the *Geometers* will carry a Man farther than all the *Dialectical* Rules. Their *Analyfis* is the proper Model we ought to form ourfelves upon, and imitate in the regular Difpofition, and gradual Progrefs, of our Inquiries ; and even he, who is ignorant of the Nature of *Mathematical Analyfis,* ufes a Method fomewhat analogous to it. The *Compofition* of the *Geometers,* or their Method of demonftrating Truths already found out, viz. *by Definitions of Words agreed upon, by felf-evident Truths, and Propofitions that have been already demonftrated,* is practicable in other Subjects, tho' not to the fame Perfection, the natural

<div align="right">Want</div>

Want of Evidence in the things themfelves not allow-
ing it; but it is imitable to a confiderable Degree.
I dare appeal to fome Writings of our own Age and
Nation, the Authors of which have been mathema-
tically inclined. I fhall add no more on this Head,
but, that one, who is accuftomed to the methodical
Syftems of Truths, which the *Geometers* have reared
up in the feveral Branches of thofe *Sciences*, which
they have cultivated, will hardly bear with the Con-
fufion and Diforder of other *Sciences*, but endeavour,
as far as he can, to reform them.

 Thirdly, *Mathematical* Knowlege adds a manly
Vigour to the Mind, frees it from *Prejudice*, *Credulity*,
and *Superftition.* This it does two Ways: 1ft, By
accuftoming us to examine, and not to take things
upon Truft. 2dly, By giving us a clear and exten-
five Knowlege of the Syftem of the World;
which, as it creates in us the moft profound Re-
verence of the almighty and wife Creator; fo it
frees us from the mean and narrow Thoughts, which
Ignorance and Superftition are apt to beget. How
great an Enemy *Mathematics* are to Superftition,
appears from this, That in thofe Countries, where
Romifh Priefts exercife their barbarous Tyranny over
the Minds of Men, *Aftronomers*, who are fully per-
fuaded of the Motion of the Earth, dare not fpeak
out: But tho' the *Inquifition* may extort a Recanta-
tion, the Pope, and a general Council too, will not
find themfelves able to perfuade to the contrary
Opinion. Perhaps, this may have given Occafion
to a calumnious Suggeftion, as if *Mathematics* were
an Enemy to Religion, which is a Scandal thrown
both on the one and the other; for Truth can never
be an Enemy to true Religion, which appears
always to the beft Advantage, when it is moft exa-
mined.

   ——— *Si propius ftes,*
  Te capiet magis. ———

             On

On the contrary, the *Mathematics* are Friends to Religion; inafmuch as they charm the Paffions, reftrain the Impetuofity of Imagination, and purge the Mind from Error and Prejudice. Vice is Error, Confufion, and falfe Reafoning; and all Truth is more or lefs oppofite to it. Befides, *Mathematical* Studies may ferve for a pleafant Entertainment for thofe Hours, which young Men are apt to throw away upon their Vices; the Delightfulnefs of them being fuch, as to make Solitude not only eafy, but defirable.

What I have faid may ferve to recommend *Mathematics* for acquiring a vigorous Conftitution of Mind; for which Purpofe they are as ufeful, as Exercife is for procuring Health and Strength to the Body. I proceed now to fhew their vaft Extent and Ufefulnefs in other Parts of Knowlege. And here it might fuffice to tell you, that *Mathematics* is the *Science* of Quantity, or the *Art* of Reafoning about things that are capable of *More* and *Lefs*; and that the moft Part of the Objects of our Knowlege are fuch; as Matter, Space, Number, Time, Motion, Gravity, &c. We have but imperfect Ideas of Things without Quantity, and as imperfect a one of Quantity itfelf without the Help of *Mathematics*. All the vifible Works of God Almighty are made in *Number*, *Weight*, and *Meafure*: Therefore, to confider them, we ought to underftand *Arithmetic*, *Geometry*, and *Statics*: And the greater Advances we make in thofe Arts, the more capable we are of confidering fuch things, as are the ordinary Objects of our Conceptions. But this will farther appear from Particulars.

And, firft, if we confider, to what Perfection we now know the Courfes, Periods, Order, Diftances, and Proportions of the feveral great Bodies of the Univerfe, at leaft, fuch as fall within our View; we fhall have Caufe to admire the Sagacity and Induftry

duſtry of the *Mathematicians*; and the Power of
Numbers and *Geometry* well applied. Let us caſt our
Eyes backward, and conſider *Aſtronomy* in its In-
fancy: Or rather let us ſuppoſe it ſtill to begin: For
Inſtance; a Colony of rude Country People, tranſ-
planted into an Iſland remote from the Commerce
of all Mankind, without ſo much as the Knowlege
of the Kalendar, and the Periods of the Seaſons,
without Inſtruments to make Obſervations, or any
the leaſt Notion of Obſervations or Inſtruments.
When is it we could expect any of their Poſterity
ſhould arrive at the Art of predicting an Eclipſe?
Not only ſo, but the Art of reckoning all
Eclipſes that are paſt or to come, for any Number
of Years? When is it we could ſuppoſe, that one
of thoſe Iſlanders, tranſported to any other Place of
the Earth, ſhould be able, by the Inſpection of the
Heavens, to find how much he were South or North,
Eaſt or Weſt of his own Iſland, and to conduct
his Ship back thither? For my Part, tho' I know
this may be and is daily done, by what is known in
Aſtronomy; yet when I conſider the vaſt Induſtry,
Sagacity, Multitude of Obſervations, and other ex-
trinſic Things neceſſary for ſuch a ſublime Piece of
Knowlege, I ſhould be apt to pronounce it impoſ-
ſible, and never to be hoped for. Now we are let
ſo much into the Knowlege of the Machine of the
Univerſe, and Motion of its Parts, by the Rules of
this *Science*, perhaps the Invention may ſeem eaſy.
But when we reflect, what Penetration and Contri-
vance were neceſſary to lay the Foundations of ſo
great and extenſive an Art, we cannot but admire
its firſt Inventors: As *Thales Mileſius*, who, as *Dio-
genes Laertius* and *Pliny* ſay, firſt predicted Eclipſes;
and his Scholar *Anaximander Mileſius*, who found
out the globous Figure of the Earth, the Equi-
noctial Points, the Obliquity of the Ecliptic, the
Principles of Gnomonics, and made the firſt Sphere

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or Image of the Heavens ; and *Pythagoras*, to whom we owe the Difcovery of the true Syftem of the World, and Order of the Planets: Though, it may be, they were affifted by the *Egyptians* and *Chaldeans*. But whoever they were, that firft made thefe bold Steps in this noble Art, they deferve the Praife and Admiration of all future Ages.

Felices animos, quibus hæc cognofcere primis,
  Inque domos fuperas fcandere cura fuit !
Credibile eft illos pariter vitiifque jocifque
  Altius humanis exferuiffe caput.
Non Venus & vinum fublimia pettora fregit ;
  Officiumve fori, militiæve labor.
Nec levis ambitio, perfufaque gloria fuco,
  Magnarumve fames follicitavit opum.
Admovere oculis diftantia fidera noftris ;
  Ætheraque ingenio fuppofuere fuo.
                              Ovid. in I<u>o</u> Faft.

But tho' the Induftry of former Ages had difco-ver'd the Periods of the great Bodies of the Uni-verfe, and the true Syftem and Order of them, and their Orbits, pretty near ; yet was there one thing ftill referved for the Glory of this Age, and the Honour of the *Englifh* Nation, the grand Secret of the whole Machine ; which, now it is difcovered, proves to be (like the other Contrivances of Infinite Wifdom) fimple and natural, depending upon the moft known and moft common Property of Matter, viz. *Gravity*. From this the incomparable Mr. *Newton* has demonftrated the Theories of all the Bodies of the Solar Syftem, of all the primary Pla-nets, and their Secondaries, and, among others, the Moon, which feem'd moft averfe to Numbers: And not only of the Planets, the floweft of which com-pletes its Period in lefs than half the Age of a Man, but likewife of the Comets, fome of which, it is

probable,

probable, fpend more than 2000 Years in one Revo-
lution about the Sun; for whofe Theory he has
laid fuch a Foundation, that after Ages, affifted
with more Obfervations, may be able to calculate
their Returns. In a Word, the Proceffion of the
Equinoctial Points, the Tides, the unequal Vibra-
tion of pendulous Bodies in different Latitudes, &c.
are no more a Queftion to thofe, that have *Geometry*
enough to underftand what he has delivered on
thofe Subjects : A Perfection in *Philofophy*, that
the boldeft Thinker durft hardly have hoped for;
and, unlefs Mankind turn barbarous, will continue
the Reputation of this Nation, as long as the Fa-
bric of Nature fhall endure. After this, what is
it we may not expect from *Geometry*, joined to *Ob-
fervations* and *Experiments?*

The next confiderable Object of Natural Know-
lege, I take to be *Light.* How unfuccefsful In-
quiries are about this glorious Body without the
Help of *Geometry*, may appear from the empty and
frivolous Difcourfes and Difputations of a Sort of
Men, that call themfelves *Philofophers*; whom no-
thing will ferve, forfooth, but the Knowlege of the
very Nature, and intimate Caufes, of every thing r
While, on the other hand, the *Geometers*, not trou-
bling themfelves with thofe fruitlefs Inquiries about
the *Nature* of *Light*, have difcovered Two remark-
able Properties of it, in the Reflexion and Refraction
of its Beams : And from thofe, and their Streight-
nefs in other Cafes, have invented the noble Arts
of *Optics, Catoptrics*, and *Dioptrics*; teaching us to
manage this fubtile Body for the Improvement
of our Knowlege, and ufeful Purpofes of Life.
They have likewife demonftrated the Caufes of
feveral celeftial Appearances, that arife from the
Inflexion of its Beams, both in the heavenly Bo-
dies themfelves, and other Phænomena, as *Parhelia*,
the *Iris*, &c. and by a late Experiment they have

difcovered

difcovered the Celerity of its Motion. And we
fhall know yet more furprizing Properties of *Light*,
when Mr. *Newton* fhall be pleafed to gratify the
World with his *Book of Light and Colours*.

The *Fluids* which involve our Earth, viz. *Air* and
Water, are the next great and confpicuous Bodies,
that Nature prefents to our View : And, I think,
we know little of either, but what is owing to *Me-
chanics* and *Geometry*. The Two chiefeft Properties
of *Air*, its Gravity, and elaftic Force, have been dif-
covered by Mechanical Experiments. From thence
the Decreafe of the Air's Denfity, according to the
Increafe of the Diftance of the Earth, has been de-
monftrated by *Geometers*, and confirmed by Experi-
ments of the Subfidence of the *Mercury* in the *Tor-
ricellian Experiment*. From this likewife, by Affift-
ance of *Geometry*, they have determined the Height
of the Atmofphere, as far as it has any fenfible Den-
fity ; which agrees exactly with another Obfervation
of the Duration of the Twilight. *Air* and *Water*
make up the Object of the *Hydroftatics*, tho' deno-
minated only from the latter, of which the Prin-
ciples were long fince fettled and demonftrated
by *Archimedes*, in his Book περὶ τῶν Ὀχυμένων,
where are demonftrated the Caufes of feveral fur-
prifing Phænomena of Nature, depending only
on the *Æquilibrium* of *Fluids*, the relative Gravities
of thefe *Fluids*, and of Solids fwimming or finking
therein. Here alfo the Mathematicians confider
the different Preffures, Refiftances, and Celerities
of Solids moved in Fluids : From whence they ex-
plain a great many Appearances of Nature, unin-
telligible to thofe who are ignorant of *Geometry*.

Next, if we defcend to the *Animal Kingdom*, there
we may fee the brighteft Strokes of Divine Me-
chanics. And whether we confider firft the *Animal
Oeconomy* in general, either in the internal Motion
and Circulation of the Juices forced through the
feveral

several Canals by the Motion of the Heart, or their external Motions, and the Inftruments wherewith thefe are performed, we muft reduce them to Me-chanical Rules, and confefs the Neceffity of the Knowlege of Mechanics to underftand them, or explain them to others. *Borelli* in his excellent Treatife *de Motu Animalium*, *Steno* in his admirable *Myologiæ Specimen*, and other Mathematical Men, on the one hand, and the nonfenfical, unintelligible Stuff that the common Writers on thefe Subjects have filled their Books with, on the other, are fuffi-cient Inftances to fhew, how neceffary *Geometry* is in fuch Speculations. The only Organ of an Ani-mal Body, whofe Structure and Manner of Opera-tion is fully underftood, has been the only one, which the *Geometers* have taken to their Share to confider. It is incredible, how fillily the greateft and ableft Phyficians talked of the Parts of the Eye and their Ufe, and of the *Modus Vifionis*, before *Kepler* by his *Geometry* found it out, and put it paft Difpute, tho' they applied themfelves parti-cularly to this, and valued themfelves on it : And *Galen* pretended a particular Divine Commiffion to treat of it. Nay, notwithftanding the full Difcovery of it, fome go on in copying their Predeceffors, and talk as *Ungeometrically* as ever. It is true, we cannot reafon fo clearly of the internal Motions of an Animal Body, as of the external, wanting fuffi-cient *Data*, and decifive Experiments : But what relates to the latter (as the Articulation, Structure, Infertion, and *Vires* of the Mufcles) is as fubject to ftrict Mathematical Difquifition, as any thing whatfoever ; and even in the Theory of Difeafes, and their Cures, thofe, who talk Mechanically talk moft intelligibly. Which may be the Reafon for the Opinion of the antient Phyficians, that Mathe-matics are neceffary for the Study of the Medicine itfelf, for which I could bring long Quotations out

of

of their Works. Among the Letters that are afcrib'd
to *Hippocrates*, there is one to his Son *Theffalus*, re‚
commending to him the Study of *Arithmetic* and
Geometry, as neceffary to Medicine. *Galen* in his
Book, intituled, ὅτι ἄειϛος ἰατρὸς καὶ Φιλόσοφ☉, begins,

Ῥοίόν τι πεπόνθασιν οἱ πολλοὶ τῶν ἀθλητῶν, ἐπιθυμῦντες μὲν
Ὀλυμπιονίκαι γενέϛχ, μηδὲν δὲ πράτ]ειν, ὡς τότε τυχεῖν,
ἐπιηδ᾽δ᾽ον]ες, ταῦτόν τι καὶ τοῖς πολλοῖς τῶν ἰαὶρῶν συμβίβη-
κεν· ἐπαινῦσι μὲν γὰρ Ἱπποκράτην, κ᾽ πρῶτον ἁπάν]ων ἡγῦν]αι·
γενέϛχ δὲ αὐτὺς ἐν ὁμοίοις ἐκείνῳ πάν]α μᾶλλον ἢ τῦτο πραΤ]υσι·
οἱ μὲν γὰρ ᾽ μικρὲν μοῖῤαν εἰς ἰαὶ᾽εικήν φησι συμβάλλεϛχ τὴν
ἀϛϱονομίαν, καὶ δ᾽ηλονότι, τὴν ταύτης ἡγεμίνην ᾽ ἀνάγκης
γεωμεϳείαν· οἱ δὲ ᾽ μόνον αὐτοὶ μεϳέρχον]αι τύτων ἰδ᾽έτεϱον,
ἀλλὰ καὶ τοῖς μεϳιῦσι μέμφον]αι. If one of the Reafons
of the Antients for this be now fomewhat un-
fafhionable, to wit, becaufe they thought a Phyfi-
cian fhould be able to know the Situation and
Afpects of the Stars, which they believed had Influ-
ence upon Men and their Difeafes, (and pofitively
to deny it, and fay, that they have none at all, is the
Effect of Want of Obfervation) we have a much
better and undoubted one in its room; *viz.* That
Mathematics are found to be the beft Inftrument of
promoting Natural Knowlege. 2dly, If we con-
fider, not only the Animal Oeconomy in general,
but likewife the wonderful Structure of the differ-
ent Sorts of Animals, according to the different
Purpofes for which they were defign'd, the various
Elements they inhabit, the feveral Ways of pro-
curing their Nourifhment, and propagating their
Kind, the different Enemies they have, and Acci-
dents they are fubject to, here is ftill a greater Need
of *Geometry*. It is pity, that the Qualities of an
expert *Anatomift*, and fkilful *Geometer*, have feldom
met in the fame Perfon. When fuch a one fhall
appear, there is a whole *Terra incognita* of delight-
ful Knowlege to employ his Time, and reward his
Induftry.

As

As for the other two Kingdoms; *Borelli*, and other *Mathematical* Men, feem to have talked very clearly of *Vegetation*: And *Steno*, another Mathematician, in his excellent Treatife *de Solido intra Solidum natu-raliter contento,* has apply'd this Part of Learning very handfomely to *Foffils*, and fome other Parts of Natural Hiftory. I fhall add only one thing more, That if we confider Motion itfelf, the great Inftru-ment of the Actions of Bodies upon one another, the Theory of it is intirely owing to the *Geometers*; who have demonftrated its Laws both in hard and elaftic Bodies; fhewed how to meafure its Quan-tity, how to compound and refolve the feveral Forces, by which Bodies are agitated, and to deter-mine the *Lines*, which thofe compound Forces make them defcribe: Of fuch Forces Gravity, being the moft conftant and uniform, affords a great Variety of ufeful Knowlege, in confidering feveral Mo-tions that happen upon the Earth; *viz.* As to the free Defcent of heavy Bodies; The Curve of Pro-jectiles; The Defcent and Weight of heavy Bodies, when they lie on inclined Planes; The Theory of the Motion of pendulous Bodies, &c.

From what I have faid, I fhall draw but one Co-rollary, That a Natural Philofopher without Mathe-matics is a very odd Sort of a Perfon, that reafons about things that have *Bulk*, *Figure*, *Motion*, *Num-ber*, *Weight*, &c. without *Arithmetic*, *Geometry*, *Me-chanics*, *Statics*, &c. I muft needs fay, I have the laft Contempt for thofe Gentlemen, that pretend to explain how the Earth was framed, and yet can hardly meafure an Acre of Ground upon the Sur-face of it: And as the Philofopher fpeaks, *Qui re-pente pedibus illotis ad Philofophos divertunt, non hoc eft fatis, quod fint omnino, ἀθεώρητοι, ἄμυσοι, ἀγεωμέτρητοι fed legem etiam dant, quâ Philofophari difcant.*

The Ufefulnefs of *Mathematics* in feveral other Arts and Sciences is fully as plain. They were looked

upon

upon by the antient Philofophers as to the Key to all Knowlege. Therefore *Plato* wrote upon his School, Οὐδεὶς ἀγεωμέτρητος εἰσίτω, *Let none unʃkilled in Geometry enter*; and *Xenocrates* told one ignorant in *Mathematics*, who defired to be his Scholar, that he was fitter to card Wool, λαβὰς γὰρ ἐκ ἔχεις Φιλοσοφίας, *You want the Handle of Philoʃophy*, viz. *Geometry.* There is no underftanding the Works of the antient Philofophers without it. *Theo. Smyrnæus* has wrote a Book, intituled, An Explanation of thofe things in Mathematics, that are neceffary for the Reading of *Plato: Ariʃtotle* illuftrates his Precepts, and other Thoughts, by Mathematical Examples; and that not only in *Logic*, &c. but even in *Ethics*, where he makes ufe of Geometrical and Arithmetical Proportion, to explain commutative and diftributive Juftice.

Every body knows, that *Chronology* and *Geography* are indifpenfable Preparations for Hiftory; a Relation of Matter of Fact being a very lifelefs infipid thing, without the Circumftances of Time and Place. Nor is it fufficient for one, that would underftand things thoroughly, that he knows the Topography, that is, the Name of the Country, where fuch a Place lies, with thofe of the near adjacent Places, and how thefe lie in refpect of one another; but it will become him likewife to underftand the fcientifical Principles of the Art: that is, to have a true Idea of a Place, we ought to know the Relation it has to any other Place, as to the Diftance and Bearing, its Climate, Heat, Cold, Length of Days, &c. which things do much enliven the Reader's Notion of the very Action itfelf. Juft fo, it is neceffary to know the technical or doctrinal Part of Chronology, if a Man would be thoroughly fkilled in Hiftory, it being impoffible, without it, to unravel the Confufion of Hiftorians. I remember Mr. *Halley* has determined

the Day and Hour of *Julius Cæsar*'s Landing in
Britain, from the Circumstances of his Relation.
And every body knows, how great Use our incom-
parable Historian Mr. *Dodwell* has made of the
calculated Times of Eclipses, for settling the Times
of great Events, which before were, as to this essen-
tial Circumstance, almost fabulous. Both *Chronology*
and *Geography*, and also the Knowlege of the Sun's
and Moon's Motions, so far as they relate to the
Constitution of the Kalendar and Year, are necessary
to a Divine ; and how sadly some otherwise Eminent
have blundered, when they meddled with things
that relate to these, and border on them, is too
apparent.

Nobody, I think, will question the Interest,
that Mathematics have in *Painting, Music*, and
Architecture, which are all founded on Numbers.
Perspective, and the Rules of Light and Shadows,
are owing to *Geometry* and *Optics :* And, I think,
those Two comprehend pretty near the whole Art
of *Painting*, except *Decorum* and *Ordinance* ; which
are only a due Observance of the History and Cir-
cumstances of the Subject you represent : For, by
Perspective, may be understood the Art of de-
signing the Outlines of your Solid, whether that
be a Building, Landskip, or Animal : And the
Draught of a Man is really as much the Perspective
of a Man, as the Draught of a Building is of a
Building ; tho', for particular Reasons, as because it
consists of more crooked Lines, &c. it is hard to
reduce the Perspective of the former, to the ordinary
established Rules.

If *Mathematics* had not reduced *Music* to a regu-
lar System, by contriving its *Scales*, it had been no
Art, but enthusiastic Rapture, left to the roving
Fancy of every Practitioner. This appears by the
extraordinary Pains, which the Antients have taken
to fit Numbers to Three Sorts of Music, the *Diato-*

nic,

ric, *Chromatic*, and *Enharmonic:* Which if we con-
fider with their Nicety in diftinguifhing their feveral
Modes, we fhall be apt to judge, they had fomething
very fine in their *Mufic*, at leaft, for moving the
Paffions with fingle Inftruments and Voices. But
Mufic had been imperfect ftill, had not *Arithmetic*
ftepped in once more, and *Guido Aretinus*, by in-
venting the Temperament, making the Fifth Falfe
by a certain determined Quantity, taught us to tune
our Organs, and intermix all the Three Kinds of
the Antients, to which we owe all the regular and
noble Harmony of our modern *Mufic*.

' As for *Civil Architecture* (of Military I fhall fpeak
afterwards) there is hardly any Part of *Mathematics*,
but is fome way fubfervient to it. *Geometry* and *Arith-
metic*, for the due Meafure of the feveral Parts of a
Building, the Plans, Models, Computation of Ma-
terials, Time, and Charges ; for ordering right its
Arches and Vaults, that they may be both firm and
beautiful : *Mechanics*, for its Strength and Firmnefs,
tranfporting and raifing Materials : And *Optics*, for
the Symmetry and Beauty. And I would not have
any affume the Character of an *Architect* without a
competent Skill in all of thefe. You fee that *Vitru-
vius* requires thefe, and many more, for making a
complete *Architect*. I muft own, that fhould any
one fet up to practife in any of the fore-mentioned
Arts, furnifhed only with his Mathematical Rules,
he would produce but very clumfy Pieces. He,
that fhould pretend to draw by the Geometrical
Rules of *Perfpective*, or compofe *Mufic* merely by
his Skill in harmonical Numbers, would fhew but
aukward Performances. In thofe compos'd Sub-
jects, befides the ftiff Rules, there muft be Fancy,
Genius, and Habit. Yet, neverthelefs, thefe Arts
owe their Being to *Mathematics*, as laying the Foun-
dation of their Theory, and affording them Pre-
cepts, which, being once invented, are fecurely rely'd
upon

upon by Practitioners. Thus many defign, that know not a Tittle of the Reafon of the Rules they practife by ; and many, no better qualify d in their way, compofe *Mufic*, better, perhaps, than he could have done, that invented the *Scale*, and the *Numbers* upon which their Harmony is founded. As *Mathematics* laid the Foundation of thefe Arts, fo they muft improve them: And he, that would invent, muft be fkill'd in Numbers: Befides, it is fit a Man fhould know the true Grounds and Reafons of what he ftudies: And he that does fo, will certainly prac-tife in his Art with greater Judgment and Variety, where the ordinary Rules fail him.

I proceed now to fhew the more immediate Ufe-fulnefs of *Mathematics* in *Civil Affairs*. To begin with *Arithmetic*, it were an endlefs Task to relate its feveral Ufes in public and private Bufinefs. The Regulation and quick Difpatch of both feem intirely owing to it. The Nations, that want it, are altoge-ther barbarous, as fome *Americans*, who can hardly reckon above Twenty. And, I believe, it would go near to ruin the Trade of the Nation, were the eafy Practice of *Arithmetic* abolifhed : For Example, were the Merchants and Tradefmen obliged to make ufe of no other than the *Roman* way of Notation by Letters, inftead of our prefent. And if we fhould feel the Want of our *Arithmetic* in the eafieft Calculations, how much more in thofe, that are fomething harder? as Intereft fimple and compound, Annuities, *&c.* in which, it is incredible, how much the ordinary Rules and Tables influence the Difpatch of Bufinefs. *Arithmetic* is not only the great Inftrument of private Commerce, but by it are (or ought to be) kept the public Accounts of a Nation: I mean thofe, that regard the whole State of a Commonwealth, as to the Number, Fructifi-cation of its People, Increafe of Stock, Improve-ment of Lands and Manufactures, Balance of

Trade,

Trade, Public Revenues, Coinage, Military Power by Sea and Land, &c. Thofe that would judge or reafon truly about the State of any Nation, muft go that way to work, fubjecting all the fore-mentioned Particulars to Calculation. This is the true *Political* Knowlege. In this refpect the Affairs of a Commonwealth differ from thofe of a private Family, only in the Greatnefs and Multitude of Particulars, that make up the Accounts. *Machiavel* goes this way to work in his Account of different Eftates. What Sir *William Petty*, and feveral others of our Countrymen, have wrote in *Political Arithmetic*, does abundantly fhew the Pleafure and Ufefulnefs of fuch Speculations. It is true, for want of good Information, their Calculations fometimes proceed upon erroneous Suppofitions: But that is not the Fault of the Art. But what is it the Government could not perform in this way, who have the Command of all public Records?

Laftly, Numbers are applicable even to fuch things, as feem to be governed by no Rule, I mean fuch as depend on *Chance*; the Quantity of Probability and Proportion of it in any Two propofed Cafes being fubject to Calculation as much as any thing elfe. Upon this depend the Principles of Game. We find Sharpers know enough of this, to cheat fome Men that would take it very ill to be thought Bubbles : And one Gamefter exceeds another, as he has a greater Sagacity and Readinefs in calculating his Probability to win or lofe in any propofed Cafe. To underftand the Theory of *Chance* thoroughly, requires a great Knowlege of Numbers, and a pretty competent one of *Algebra*.

The feveral Ufes of *Geometry* are not much fewer than thofe of *Arithmetic*. It is neceffary for afcertaining of Property both in Planes and Solids, or in Surveying and Gauging. By it, Land is fold by the Meafure, as well as Cloth : Workmen are paid

the

the due Price of their Labour, according to fuperficial or folid Meafure of their Work : And the Quantity of Liquors determined for a due Regulation of their Price and Duty. All which do won-derfully conduce to the eafy Difpatch of Bufinefs, and the preventing of Frauds and Controverfies. I need not mention the meafuring Diftances, laying down of Plans and Maps of Countries, in which we have daily Experience of its Ufefulnefs. Thefe are fome familiar Inftances of things, to which *Geometry* is ordinarily applied : Of its Ufe in *Civil*, *Military*, and *Naval Architecture*, we fhall fpeak afterwards.

From *Aftronomy* we have the regular Difpofition of our Time, in a due Succeffion of Years, which are kept within their Limits as to the Return of the Seafons, and the Motion of the Sun. This is no fmall Advantage for the due Repetition of the fame Work, Labour, and Actions. For many of our Public, Private, Military, and Country Affairs, Appointments, *&c.* depending on the Products of the Ground, and they on the Seafons ; it is neceffary, that the Returns of them be adjufted pretty near to the Motion of the Sun : And we fhould quickly find the Inconveniency of a *vague* undetermined Year, if we ufed that of the *Mahumetans*, whofe Beginning, and every Month, wanders through all the Days of ours or the Solar Year, which fhews the Seafons. · Befide, the adjufting of the Moon's Motion to the Sun's is required for the decent Obfervation and Celebration of the *Church-Feafts* and *Fafts*, according to the antient Cuftom, and primitive Inftitution ; and, likewife, for the knowing of the Ebbing and Flowing of the Tides, the Spring, and Neap Tides, Currents, *&c.* So that whatever fome People may think of an *Almanac* where all thefe are fet down, it is oftentimes the moft ufeful Paper that is publifhed the fame Year with it : Nay, the

Nation

Nation could better fpare all the voluminous Au-
thors in the *Term-Catalogue*, than that fingle Sheet.
Befides, without a regular Chronology, there can
be no certain Hiftory; which appears by the Con-
fufion amongft Hiftorians before the right Difpofi-
tion of the Year, and, at prefent, among the *Turks*,
who have the fame Confufion in their Hiftory as in
their Kalendar. Therefore, a Matter of fuch Im-
portance might well deferve the Care of the *Great
Emperor*, to whom we owe our prefent Kalendar;
who was himfelf a great Proficient in *Aftronomy*.
Pliny has quoted feveral things from his Books of
the *Rifing* and *Setting* of the Stars, *Lib.* XVIII. *cap.*
25, 26, &c. and *Lucan* makes him fay,

———*Media inter prælia femper*
Stellarum, cælique plagis, fuperifque vacavi.

The *Mechanics* have produced fo many ufeful
Engines, fubfervient to Conveniency, that it would
be a Tafk too great to relate the feveral Sorts of
them: Some of them keep Life itfelf from being
a Burden. If we confider fuch, as are invented
for raifing Weights, and are employed in Building,
and other great Works, in which no Impediment
is too great for them; or *Hydraulic* Engines for
raifing of Water, ferving for great Ufe and Com-
fort to Mankind, where they have no other way to
be fupply'd readily with that neceffary Element; or
fuch as, by making Wind and Water work for
us, fave animal Force, and great Charges, and per-
form thofe Actions, which require a vaft Multitude
of Hands, and without which every Man's Time
would be too little to prepare his own Aliment, and
other Neceffaries; or thofe Machines, that have
been invented by Mankind for Delight and Curio-
fity, imitating the Motions of Animals, or other
Works of Nature; we fhall have Reafon to admire
and extol fo excellent an Art. What fhall we fay
of

of the feveral Inftruments, which are contrived to meafure Time? We fhould quickly find the Value of them, if we were reduced to the Condition of thofe barbarous Nations that want them. The *Pendulum Clock*, invented and completed by that famous *Mathematician* Monfieur *Huygens*, is an ufeful Invention. Is there any thing more wonderful, than feveral *Planetary Machines*, which have been invented to fhew the Motions of the heavenly Bodies, and their Places at any time? Of which the moft ingenious, according to the exacteft Numbers, and true Syftem, was made by the fame M. *Huygens:* To which we may very juftly apply *Claudian*'s noble Verfes upon that of *Archimedes:*

Jupiter in parvo cùm cerneret æthera vitro,
    Rifit, & ad fuperos talia dicta dedit:
Huccine mortalis progreffa potentia curæ?
    Jam meus in fragili luditur orbe labor.
Jura poli, rerumque fidem, legefque Deorum
    Ecce Syracufius tranftulit arte fenex.
Inclufus variis famulatur fpiritus aftris,
    Et vivum certis motibus urget opus.
Percurrit proprium mentitus Signifer annum,
    Et fimulata novo Cynthia menfe redit.
Jamq; fuum volvens audax induftria mundum
    Gaudet, & humanâ fidera mente regit.
Quid falfo infontem tonitru Salmonea miror?
    Æmula naturæ parva reperta manus.

Here I ought to mention the *Sciatherical* Inftruments, for want of which there was a time, when the *Grecians* themfelves were forced to meafure the Shadow, in order to know the Hour; and, as *Pliny* (*cap. ult. lib.* VII.) tells us, the *Romans* made ufe of an erroneous Sun-dial for Ninety-nine Years, till *Q. Marcius Philippus*, their Cenfor, fet up a better; which, no doubt, at that time, was thought a

Jewel.

Jewel. And, at laft, that famous Pyramid was fet up in the *Campus Martius*, to ferve for a Gnomon to a Dial marked on the Street. To this Sort of Engines ought to be referred *Spheres, Globes, Aftrolabes, Projections of the Sphere,* &c. Thefe are fuch ufeful and neceffary things, that alone may recommend the Art, by which they are made. For, by thefe, we are able in our Clofet to judge of the celeftial Motions, and to vifit the moft diftant Places of the Earth, without the Fatigue and Danger of Voyages ; to determine concerning their Diftance, Situation, Climate, Nature of the Seafons, Length of their Days, and their Relation to the celeftial Bodies, as much as if we were Inhabitants. To all thefe I might add thofe Inftruments, which the *Mathematicians* have invented to execute their own Precepts, for making *Obfervations* either by Sea or Land, *Surveying, Gauging,* &c.

The *Catoptrics* and *Dioptrics* furnifh us with Variety of ufeful Inventions, both for the promoting of Knowlege, and the Conveniencies of Life ; whereby Sight, the great Inftrument of our Perception, is fo much improved, that neither the Diftance nor the Minutenefs of the Object are any more Impediments to it. The *Telefcope* is of fo vaft Ufe, that, befides the delightful and ufeful Purpofes it is apply'd to here below, as the defcrying Ships, and Men, and Armies, at a Diftance, we have, by its means, difcovered new Parts of the Creation, frefh Inftances of the furprifing Wifdom of the adorable Creator. We have, by it, difcovered the *Satellites* of *Jupiter*, the *Satellites* and *Ring* of *Saturn*, the Rotation of the Planets about their own Axes ; befides other Appearances, whereby the Syftem of the World is made plain to *Senfe*, as was before to *Reafon*. The *Telefcope* has alfo improved the Manner of *Aftronomical Obfervations*, and made them much more accurate, than it was poffible for them

to

to be before. And thefe Improvements in *Aftronomy*, have brought along with them (as ever) corre-, fpondent Improvements in *Geography*. From the. Obfervation of *Jupiter's Satellites*, we have a ready Way to determine the Longitude of Places on the, Earth. On the other hand, the *Microfcope* has not, been lefs ufeful in helping us to the Sight of fuch. Objeſts, as by their Minutenefs efcape our naked Eye. By it Men have purfued Nature into its moſt retired Receſſes; fo that now it can hardly any more hide its greateſt Myſteries from us. How much have we learned by the Help of the *Micro- fcope* of the Contrivance and Struſture of animal, and vegetable Bodies, and the Compoſition of Fluids and Solids? But if thefe *Sciences* had never gone further, than by their ſingle *Specula* and *Lentes* to give thofe furprifing Appearances of Objeſts, and their Images, and to produce Heat unimitable by our hotteſt Furnaces, and to furniſh infallible, eaſy, cheap, and fafe Remedies for the Decay of our Sight ariſing commonly from old Age, and for Purblindnefs, they had merited the greateſt Eſteem, and invited to the clofeſt Study: Efpecially, if we confider, that fuch as naturally are almoſt blind, and either know not their neareſt Acquaintance at the Diftance of a Room's Breadth, or cannot read, in order to pafs their Time pleafantly, are, by Glaſſes adapted to the Defeſt of their Eyes, fet on a Le-vel again with thofe that enjoy their Eye-fight beſt, and that without Danger, Pain, or Charge.

Again, *Mathematics* are highly ferviceable to a Nation in *Military Affairs*. I believe this will be readily acknowleged by every Body. The Affairs of War take in Number, Space, Force, Diftance, Time, &c. (Things of *Mathematical* Confideration) in all its Parts, in *Taſtics*, *Caſtrametation*, *Fortifying*, *Attacking*, and *Defending*. The Antients had more Occafion for Mechanics in the Art of War than we

D   have;

have; Gunpowder readily producing a Force far
exceeding all the Engines, they had contriv'd for
Battery. And this, I reckon, has loft us a good
Occasion of improving our Mechanics; the Cun-
ning of Mankind never exerting itself so much, as
in their Arts of destroying one another. But, as
Gunpowder has made Mechanics less serviceable
to War; it has made *Geometry* more necessary:
There being a Force or Resistance in the due Mea-
sures and Proportions of the Lines and Angles of
a Fortification, which contribute much towards
its Strength. This Art of *Fortification* has been
much study'd of late, but I dare not affirm, that
it has attained its utmost Perfection. And tho',
where the Ground is regular, it admits but of small
Variety, the Measures being pretty well determined
by *Geometry* and Experience, yet where the Ground
is made up of natural *Strengths* and *Weaknesses*, it
affords some Scope for Thinking and Contrivance.
But there is another much harder Piece of *Geometry*,
which Gunpowder has given us Occasion to im-
prove, and that is the Doctrine of Projectiles;
whereon the Art of *Geometry* is founded. Here the
Geometers have invented a beautiful Theory, and
Rules and Instruments, which have reduced the
Casting of Bombs to great Exactness. As for
Tactics and *Castrametation*, *Mathematics* retain the
same Place in them as ever. And some tolerable
Skill in these is necessary for *Officers*, as well as for
Engineers. An *Officer*, that understands Fortifica-
tion, will, *cæteris paribus*, much better defend his
Post, as knowing, wherein its Strength consists, or
make use of his Advantage to his Enemy's Ruin,
than he that does not He knows, when he leads
never so small a Party, what his Advantages and
Disadvantages in Defending and Attacking are, how
to make the best of his Ground, &c. And hereby
can do truly more Service than another of as much

3   Courage,

Courage, who, for want of such Knowlege, it may be, throws away himself, and a Number of brave Fellows under his Command ; and it is well, if the Mischief reaches no further. As for a competent Skill in *Numbers*, it is so necessary to *Officers*, that no Man can be safely trusted with a Company, that has it not. All the Business is not to fire Musquets ; the managing of Affairs, the dealing with Agents, *&c.* happen more frequently. And the higher the Command is, the more Skill in all the aforesaid things is required. And I dare appeal to all the Nations in *Europe*, whether, *cæteris paribus*, Officers are not advanced in Proportion to their Skill in *Mathematical Learning* ; except, that sometimes *Great Names* and *Quality* carry it ; but still so, as that the Prince depends upon a Man of *Mathematical Learning*, that is put as Director to the *Quality*, when that Learning is wanting in it.

Lastly, *Navigation*, which is made up of *Astronomy* and *Geometry*, is so noble an Art, and to which Mankind owes so many Advantages, that, upon this single Account, those excellent Sciences deserve most of all to be studied, and merit the greatest Encouragement from a Nation, that owes to it both its Riches and Security. And not only does the common Art of *Navigation* depend on *Mathematics*, but whatever Improvements shall be made in the *Architectura Navalis*, or Building of Ships, whether they are designed for Merchant-Ships, or Ships of War, whether swift running, or bearing a great Sail, or lying near the Wind, be desired, these must all be the Improvements of *Geometry*. Ship-Carpenters, indeed, are very industrious ; but in these things they acknowlege their Inability, confess that their best Productions are the Effects of Chance, and implore the *Geometer*'s Help. Nor will common *Geometry* do the Business ; it requires the most abstruse to determine the different Sections of

a Ship,

a Ship, according as it is defigned for any of the aforefaid Ends. A *French* Mathematician *P. le Hofte* has lately endeavoured fomething in this way : And tho' it is not free from Errors, as requiring a fuller Knowlege in *Geometry*; yet is the Author much to be commended for this, as having bravely defigned, and pav'd the Way for other Mathematicians; and alfo for the former and bigger Part of his Book, wherein he brings to a Syftem the Working of Ships, and the *Naval Tactics*, or the regular Difpofition of a Fleet in Attacking, Fighting, and Retreating, according to the different Circumftances of Wind, Tides, &c.

The great Objection, that is made againft the Neceffity of *Mathematics*, in the fore-mentioned great Affairs of *Navigation*, the Art *Military*, &c. is, that we fee thofe Affairs are carried on and managed by fuch, as are not great Mathematicians; as Seamen, Engineers, Surveyors, Gaugers, Clockmakers, Glafs-grinders, &c. and that the Mathematicians are commonly fpeculative retired, ftudious Men, that are not for an active Life and Bufinefs, but content themfelves to fit in their Studies, and pore over a *Scheme*, or a *Calculation*. To which there is this plain and eafy Anfwer: The Mamaticians have not only invented and ordered all the Arts above-mentioned, by which thofe grand Affairs are managed; but have laid down Precepts, contrived Inftruments and Abridgments fo plainly, that common Artificers are capable of practifing by them, tho' they underftand not a Tittle of the Grounds, on which the Precepts are built. And in this they have confulted the Good and Neceffities of Mankind. Thofe Affairs demand fo great a Number of People to manage them, that it is impoffible to breed fo many good or even tolerable Mathematicians. The only thing then to be done was to make their Precepts fo plain, that they might

be

be underftood and practifed by a Multitude of Men. This will beft appear by Examples. Nothing is more ordinary than Difpatch of Bufinefs by common *Arithmetic*, by the *Tables of fimple and compound Intereft, Annuities,* &c. Yet how few Men of Bufinefs underftand the Reafons of common *Arithmetic,* or the Contrivance of thofe Tables, now they are made; but fecurely rely on them as true. They were the good and the thorough Mathematicians, that made thofe Precepts fo plain, and calculated thofe Tables, that facilitate the Practice fo much. Nothing is more univerfally neceffary, than the meafuring of Plains and Solids: And it is impoffible to breed fo many good Mathematicians, as that there may be one, that underftands all the *Geometry* requifite for Surveying, and Meafuring of *Prifms* and *Pyramids*, and their Parts, and meafuring *Fruftums* of *Conoids* and *Spheroids*, in every Market-Town, where fuch Work is neceffary: The Mathematicians have therefore infcrib'd fuch Lines on their common Rulers, and Slipping Rulers, and adapted fo plain Precepts to them, that every Country-Carpenter, and Gauger, can do the Bufinefs accurately enough; though he knows no more of thofe Inftruments, Tables, and Precepts he makes ufe of, than a Hobby-horfe. So in *Navigation*, it is impoffible to breed fo many good Mathematicians, as would be neceffary to fail the hundredth Part of the Ships of the Nation. But the Mathematicians have laid down fo plain and diftinct Precepts, calculated neceffary Tables, and contrived convenient Inftruments, fo that a Sea-man, that knows not the Truths, on which his Precepts and Tables depend, may practife fafely by them. They *refolve* Triangles every Day, that know not the Reafon of any one of their *Operations.* Seamen in their

Calcu-

Calculations make use of *artificial Numbers*, or *Logarithms*, that know nothing of their Contrivance : And indeed all those great Inventions of the most famous Mathematicians had been almost useless for those common and great Affairs, had not the Practice of them been made easy to those who cannot understand them. From hence it is plain, that it is to those *Speculative Retir'd Men*, we owe the Rules, the Instruments, the Precepts for using them, and the Tables which facilitate the Dispatch of so many great Affairs, and supply Mankind with so many Conveniencies of Life. They were the Men, that taught the World to apply *Arithmetic, Astronomy,* and *Geometry,* to *Sailing,* without which the Needle would be still useless. Just the same way in the other Parts of *Mathematics,* the Precepts that are practised by Multitudes, without being understood, were contriv'd by some few great *Mathematicians.*

Since then it has been shewn, how much *Mathematics* improve the Mind, how subservient they are to other Arts, and how immediately useful to the Commonwealth, there needs no other Arguments or Motives to a Government, to encourage them. This is the natural Conclusion from these Premises. *Plato*, in his *Republic, (lib.* VII.) takes care, *That, whoever is to be educated for Magistracy, or any confiderable Post in the Commonwealth, may be instructed first in Arithmetic, then in Geometry, and thirdly in Astronomy.* And however necessary those Arts were in *Plato's* time, they are much more so now: The Arts of War and Trade requiring much more the Assistance of those *Sciences* now, than they did then; as being brought to a greater Height and Perfection. And accordingly we see, these *Sciences* are the particular Care of Princes, that design to raise the

Force and Power of their Countries. It is well
known, that this is none of the leaft Arts, where-
by the *French King* has brought his Subjects to
make that Figure at Sea, which they at this
Time do; I mean, the Care He takes for Educa-
tsng thofe appointed for Sea-fervice in *Mathema-
tical Learning*. For in the *Ordonnance Marine*,
Title VIII. ' He orders, that there be Profeffors
' to teach Navigation publickly in all the Sea-port
' Towns, who muft know *Defigning*, and teach it
' to their Scholars, in order to lay down the Ap-
' pearances of Coafts, *&c.* They are to keep
' their Schools open, and read four times a Week
' to the Seamen, where they muft have Charts,
' Globes, Spheres, Compaffes, Quadrants, Aftro-
' labes, and all Books and Inftruments neceffary
' to teach their Art. The Directors of Hofpitals
' are obliged to fend thither yearly two or three
' of their Boys to be taught, and to furnifh them
' with Books and Inftruments. Thofe Profeffors
' are oblig'd to examine the Journals depofited
' in the Office of Admiralty, in the Place of their
' Eftablifhment; to correct the Errors in Prefence
' of the Seamen, and to reftore them within a
' Month, ' *&c.* King *Charles* the Second, who well
underftood the Importance of Eftablifhments of
this Nature, founded one fuch School in *Chrift's
Hofpital, London*; which, I believe, is inferior to
none of the *French:* But 'tis to be wifhed there
were many more fuch. His prefent *Majefty*, during
the Time of the late War, eftablifhed a *Mathemati-
cal Lecture* to breed up Engineers and Officers, as
knowing very well the Importance thereof. And
this continued fome time after the *Peace*. And it
is worthy the Confideration of the *Wifdom* of the
Nation, whether the reftoring and continuing this,
even in *Peace*, be not expedient for the breeding
of

of Engineers, who are fo ufeful and valuable, and fo difficult to be had in Time of War, and fo little dangeous in Times of *Peace.*

Befides the Crowd of *Merchants, Seamen, Surveyors, Engineers, Ship-carpenters, Artifans,* &c. that are to be inftructed in the Practice of fuch Parts of *Mathematics,* as are neceffary to their own Bufinefs refpectively, a competent Number of *able Mathematicians* ought to be entertained, in order to apply themfelves to the Practice ; not only to inftruct the former Sort, but likewife to remove thofe Obftacles, which fuch, as do not think beyond their common Rules, cannot overcome. And no doubt it is no fmall Impediment to the Advancement of Arts, that *Speculative Men,* and *good Mathematicians,* are unacquainted with their particular Defects, and the feveral Circumftances in them, that render things *practicable* or *impracticable.* But if there were public Encouragement, we fhould have skilful Mathematicians employed in thofe Arts, who would certainly find out and remedy the Imperfections of them. The prefent Lords Commiffioners of the Admiralty, knowing that there are ftill two great *Defiderata* in Navigation, to wit, *The Theory of the Variation of the magnetical Needle,* and a *Method of finding out the Longitude of any Place,* that may be practicable at Sea by Seamen, and being fenfible, of what Importance it would be to find out either of them, have employed a very fit Perfon, the ingenious Mr. *Halley,* who has joined an intire Acquaintance in the Practice, to a full and thorough Knowlege of the more abftrufe Parts of *Mathematics.* And now that he is returned, it is not doubted, but he will fatisfy thofe that fent him, and, in due time, the World too, with his Difcoveries in both thofe Particulars, and in

many

many other that he has had Occafion to make. And where a long Series of Obfervations and Experiments is neceffary, he has, no doubt, laid fuch a Foundation, as that After-Obfervers may gradually perfect them. If it were not for more than the Correcting the Situation of the Coafts where he touched, and by them others, whofe Relation to the former is known, the Nation is more than triply paid: And thofe who fent him, have, by this Miffion, fecured to themfelves more true Honour, and lafting Fame, than by Actions, that, at firft View, appear more magnificent.

The next thing that is neceffary for the Improvement of *Mathematical Learning*, is, That Mathematics be more generally ftudied at our *Univerfities* than hitherto they have been. From thofe Seminaries the State juftly expects and demands Thofe who are acquainted both with the *Speculation* and *Practice*. In thofe are all the Encouragements to them imaginable, Leifure and Affiftance. There are ftill at hand Books and Inftruments; as alfo other Scholars that have made equal Progrefs, and may be Comrades in Study; and the Direction of the Profeffors. There are alfo in Perfection all the Incitements to this Study; and efpecially an Acquaintance with the Works of the Antients, where this *Learning* is fo much recommended. There other Faculties are ftudied, to which it is fubfervient. There alfo are the Nobility and Gentry bred; who, in due time, muft be called to their Share in the Government of the *Fleets, Army, Treafury*, and other publ'c Employments, where *Mathematical Learning* is abfolutely neceffary, and, without which, they, tho' of never fo great natural Parts, muft be at the Mercy and Difcretion of their Servants and Deputies;

E puties;

puties ; who will firſt cheat them, and then laugh
at them. And not only public Employments,
but their private Concerns, demand Mathema-
tical Knowlege. If their Fortunes lie in Woods,
Coal, Salt, Manufactures, &c. the Neceſſity of
this Knowlege is open and known : And, even in
Land-Eſtates, no Undertaking for Improvement
can be ſecurely relied upon without it. It not
only makes a Man of Quality and Eſtate his
whole Life more illuſtrious, and more uſeful for
all Affairs, (as *Hippocrates* ſays, Ἰσοξίης δὲ μελέτω σοὶ,
ὦ παῖ, Γεωμέ]ρικῆς, καὶ Ἀριθμήσι④· ἡ γὰρ μόνον σέο κϳ τὸν
βίον εὐκλέα κϳ ἐπὶ πολλά χρήσιμον ἐς ἀνθρωπίνην μοίρην
ἐπι]ελέσει, ἀλλά κϳ τὴν ψυχὴν ὀξυῖέρην τε κϳ τηλαυξε-
ςέρην, &c.) but in particular, it is the beſt Com-
panion for a Country Life. Were this once be-
come a faſhionable Study, (and the *Mode* exerciſes
its Empire over Learning as well as other things)
it is hard to tell, how far it might influence the
Morals of our Nobility and Gentry, in rendering
them ſerious, diligent, curious ; taking them off
from the more fruitleſs and airy Exerciſes of the
Fancy, which they are apt to run into.

The only Objection I can think of, that is
brought againſt theſe Studies, is, That Mathema-
tics require a particular Turn of Head, and a
happy Genius, that few People are Maſters of ;
without which all the Pains beſtowed upon the
Study of them are in vain : They imagine, that a
Man muſt be born a Mathematician. I anſwer, That
this *Exception* is common to Mathematics and
other Arts. That there are Perſons that have a
particular Capacity and Fitneſs to one more than
another, every body owns : And, from Experience,
I dare ſay, it is not in any higher Degree true
concerning Mathematics, than the others. A Man
of good Senſe and Application is the Perſon that

is

is by Nature fitted for them ; efpecially if he be-
gins betimes : And, if his Circumftances have been
fuch, that this did not happen, by prudent Di-
rection the Defect may be fupplied, as much as in
any Art whatfoever. The only Advantage this
Objection has, is, That it is on the Side of Soft-
nefs and Idlenefs, thofe powerful Allies!

There is nothing further remains, Sir, but that
I give you my Thoughts in general concerning
the *Order* and *Method* of ftudying *Mathematics* ;
which I fhall do very briefly, as knowing that you
are already acquainted with the beft Methods; and
others with you may have them eafily from the
beft and ableft Hands.

Firft, then, I lay down for a Principle, That no-
body at an *Univerfity* is to be taught the Practice
of any Rule without the true and folid Reafon and
Demonftration of the fame. Rules without De-
monftration muft and ought to be taught to *Sea-
men, Artifans,* &c. as I have already faid ; and
Schools for fuch People are fit in Sea-ports and
Trading-Towns ; but it is far below the Dignity
of an *Univerfity,* which is defign'd for folid and
true Learning, to do this. It is from the Uni- .
verfities that they muft come, who are able to
remedy the Defects of the Arts ; and therefore
nothing muft be taken on Truft there. *Seamen*
and *Surveyors,* &c. remember their Rules, becaufe
they are perpetually practifing them ; but *Scholars,*
who are not thus employ'd, if they know not the
Demonftration of them, prefently forget them.

Secondly, No Part of Mathematics ought to
be taught by *Compendiums.* This follows from the
former. *Compendiums* are fit to give a general and
fuperficial Knowlege, not a thorough one. It is
Time, and not the Bulk of Books, we ought to
be fparing of : And I appeal to any Perfon of

Experience,

Experience, whether folid Knowlege is not ac-
quir'd in fhorter time by Books treating fully of
their Subjeits, than by Compendiums and Abridg-
ments.

From hence it follows, that the *Elements* of
Arithmetic and *Geometry* are to be taught. *Euclid*,
in his Thirteen Books of Elements, gives us both :
but our prefent Way of Notation fuperfedes fome
of thofe of *Arithmetic*, as demonftrating the Rules
from the Operations themfelves. There remain
then the firft Six Books for the *Geometry* of *Planes*,
and the laft Three for *Stereometry*. The reft ought
to be read in their own Place, for the Perfeition
of *Arithmetic*. In teaching thefe, Care ought to
be taken to make ufe of fuch Examples, as fuit
with the Condition of the Scholar: For Inftance,
Merchants Accompts and *Affairs* for Examples of the
Operations of *Arithmetic*, to one that is afterwards
to have a Concern that way ; whereas, to a Man
of the firft Quality, Examples from the *Increafe*
and *Decreafe* of the *People*, or from *Land* or *Sea
Force*, and from the *Taitics*, ought to be propofed.
For, it is certain, nothing makes one tir'd fooner,
than the frivolous and trifling Examples, that are
commonly brought for the Exercife of the Rules
of *Arithmetic* and *Geometry* ; tho' this is common to
them with the other Arts, as *Grammar*, *Logic*, &c.

The Manner of Writing of the Mathematicians of
This and the former Age makes *Trigonometry*, with
the Manner of Conftruiting its Tables, *&c.* almoft
Elementary : And the *Praitical Geometry*, commonly
fo call'd, is very fit to come next, as an elegant
Application of the *Elements* of *Geometry* to Bufinefs,
as *Surveying*, *Gauging*, &c.

After the Elements of *Spherics*, which are per-
feitly well-handled by *Theodofius*, a full Infight into
the Principles of *Aftronomy* will be neceffary.

Mechanics

Mechanics come next to be read, which are the Ground of a great Part of natural Learning ; and, afterwards, *Optics, Catoptrics,* and *Dioptrics.*

But none of thefe, except the Elements, can be fully underftood, until one is pretty well skill'd in *Conic Sections:* And all thefe are made more eafy by fome tolerable Skill in *Algebra,* and its Application to *Geometry.*

Thefe Foundations being laid, any one may, with great Eafe, purfue the Study of the Mathematics, as his Occafions require ; either in its abftract Parts, and the more *recondite Geometry,* and its Application to Natural Knowlege ; or in Mechanics ; by profecuting the *Statics, Hydroftatics, Balliftics,* &c. : Or in *Aftronomy,* by its Application to *Geography, Navigation, Gnomonics, Aftrolabes,* &c. But, in moft of thefe, a particular Order is not neceffary : Any one may take That firft, which he is moft inclined to.

I fhall not offer you any Advice concerning the Choice of Books ; but refer you (if you want any) to the Direction of thofe who are eminent among you in this Part of Learning. I ask your Pardon for the Omiffion of *Ceremony* in thefe Papers ; having followed rather the ordinary Way of *Effay,* than *Letter.* And, wifhing you good Succefs in your Studies, I am,

S I R,

Your Friend and Servant.

25 *Novemb.*
1700.

F I N I S.